# Barter Market Place Model

*openbarter*

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Internet that reduced significantly transaction cost of exchanges, is used here for multilateral barter exchanges. The model presented here fosters cooperation as much a competition. It proposes a compromise between the traditional opposition between planned and liberal economy, allowing free trade and, at the same time, necessary rationing.

## 1 Context

Barter is an exchange of ownership of **goods** between *at least* two **participants**, each providing a good to an other participant, for an other good he receives in exchange. It is the most general form of economic exchange.

Barter can have diverse forms. It is **bilateral** when it occurs between two participants and **non-bilateral** between at least three participants. The exchange can be immediate or delayed, depending of future events.

We will only consider here multilateral barter with immediate exchange of property, where exchanges can be seen as cycles, and where exchanged goods can be described by a quantity and a measurement standard defining it's **quality**.

We use to see barter market as less liquid than a market using money, because it is not easy to find someone that accepts what we provide and, at the same time, provides what we are looking for. But when non-bilateral exchanges are considered, barter is nearly as liquid as money when the diversity of qualities is small compared to the number of bids; this condition being verified for the most essential qualities exchanged.

## 2 Model

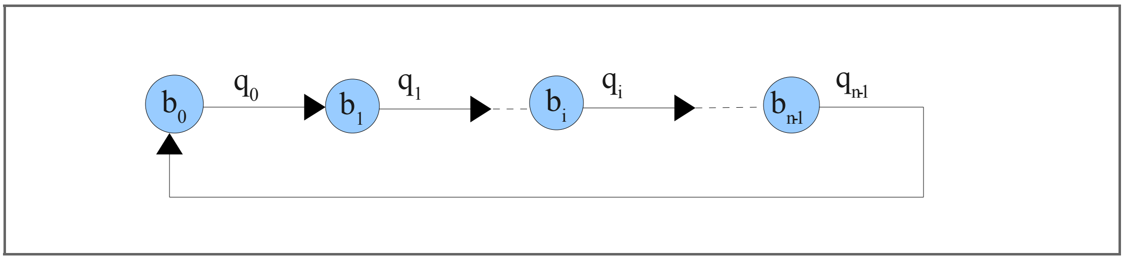
A **stock** is a good defined by a quantity and a quality standard; the good is owned, measurable and severable.

A **bid** is an unilateral commitment of the owner to give his stock or a part of it in exchange of a good of another quality. This bid is defined by the stock, a ratio $\omega$ between provided and received quantities and the quality required in exchange. $\omega$ is a different way to express the price without using money.

### 2.1 Conditions for an agreement to be possible

We will determine relations that should exist between bids for an exchange to be possible. By agreement, we mean a contract formed by meeting bids of participants, where each partner provides a quality of good an other requests, the relation between bids forming an exchange cycle.

For the quantitative aspect; we consider a cycle of n bids where each bid i provides a quantity $q_i$, and where i belongs to [0,n[.




The $\omega_i$ of each bid $b_i$ should be such that:

$$\omega_i = \frac{q_i}{q_{i-1}}$$

except for $\omega_0$ where we have:

$$\omega_0 = \frac{q_0}{q_{n-1}}$$

We have hence these relations:

$$\omega_{n-1} = \frac{q_{n-1}}{q_{n-2}}$$

$$\omega_i = \frac{q_i}{q_{i-1}} \quad , \quad i \in [1, n-2]$$

$$\omega_0 = \frac{q_0}{q_{n-1}}$$

The product of these equalities gives:

$$\prod_{i=0}^{n-1} \omega_i = \frac{q_{n-1}}{q_{n-2}} . \prod_{i=1}^{n-2} \frac{q_i}{q_{i-1}} . \frac{q_0}{q_{n-1}} = \frac{\prod_{i=0}^{n-1} q_i}{\prod_{i=0}^{n-1} q_i} = 1$$

Hence, a necessary condition for an agreement to be formed is that the product of the $\omega_i$ equals 1.

$$\prod_{i=0}^{n-1} \omega_i = 1$$

We note Ω the product of $\omega_i$.

In other words; for an agreement to be possible between bids, two conditions are necessary: the first, qualitative, is that each bid provides the quality an other bid requests; the other, quantitative, is that $\Omega = 1$.

But by meeting bids of the cycle they form, the product of their $\omega_i$ is not generally equal to 1, and $\omega_i$ need to be adjusted to form an agreement.

## 2.2 Fair compromise

A barter is figured out when each participant estimates for what he receives a *subjective* value greater than that for what he gives away. When the difference between theses values of goods received and given away – that motivates each participant to contribute to the barter – is appreciated equally by participants, the barter is well balanced.

The model considers the barter is **fair** when the adjustment is well shared between partners. More precisely, $\omega_i$ is adjusted to a value $\omega_i'$ by increasing or decreasing it so that $\omega_i' = k.\omega_i$ , k being the same for all partners i.

Since $\omega_i'$ allow the formation of the agreement, they verify the relation:

$$\prod_{i=0}^{n-1} \omega_i' = 1$$

By combining both equalities,

we have:

$$\prod_{i=0}^{n-1} k.\omega_i = 1$$

Using $\Omega$, the product of $\omega_i$ , we obtain :

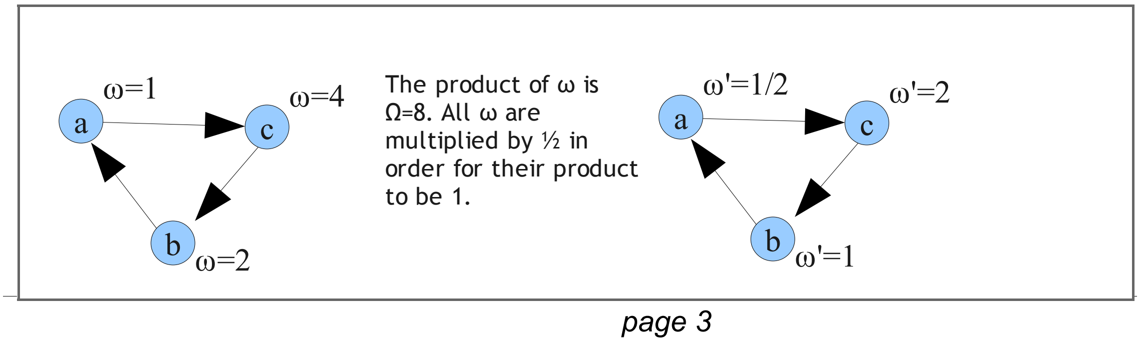
[a]

$$\omega_i' = \omega_i . \Omega^{-\frac{1}{n}}$$

$\Omega$ is a non dimensional quantity we call the **collective product** of the cycle of bids. It is greater than 1 if bids of the cycle are collectively more generous than necessary; the adjustment then represents a profit for partners. On the contrary, when this product $\Omega$ is less than 1, an effort is required from them to obtain a compromise.

This adjustment of $\omega$ represents of the bargaining of barter. For bilateral transactions using money, it is equivalent to the convergence of buyer an seller prices.

The figure shows an example of an adjustment: the left side represents the cycle before the adjustment, and the right side represents it after.



When Ω<1, we call the effort k>1 shared between partners a **collective loss**. Likewise, when Ω>1, we call the profit k<1 a **collective profit**.

### Special case of money

For a bilateral transaction using money, we just consider the monetary standard as a ordinary distinct quality. The price of the buyer is $p_b$, and the one of the seller is $p_s$ . Using ω notation, we have $\omega_b=p_b$ and $\omega_s=1/p_s$ since ω is the ratio between provided and required quantities.

We see that $\omega_b.\omega_s=1$ and $p_b=p_s$ are equivalent statement.

It is easy to show that the adjustment of ω described earlier is the geometric mean of buyer and seller prices[1].

The market competition is performed by choosing the best price.

From the buyer's point of view, it is the lowest prices between offers of sellers.

From the seller's point of view, it is the highest between offers of buyers.

Expressed using Ω , the market competition is the choice of the greatest Ω in both cases.

## 2.3 Non-bilateral competition

The model simply extends the competition rule maximizing Ω to the non-bilateral case. This rule minimizes the collective loss, and maximises the collective profit.

But other rules could also be defined whose projection on the bilateral case is the choice of best price.

A typical example is the rule is the following:

If the interest of partners is to receive the maximum while giving the minimum – or simply minimize ω' , the [a] formula shows that $\Omega^{-\frac{1}{n}}$ should be minimum, or $\Omega^{\frac{1}{n}}=\Delta$ maximum. The rule is here that Δ is maximized. We call Δ the **individual product**.

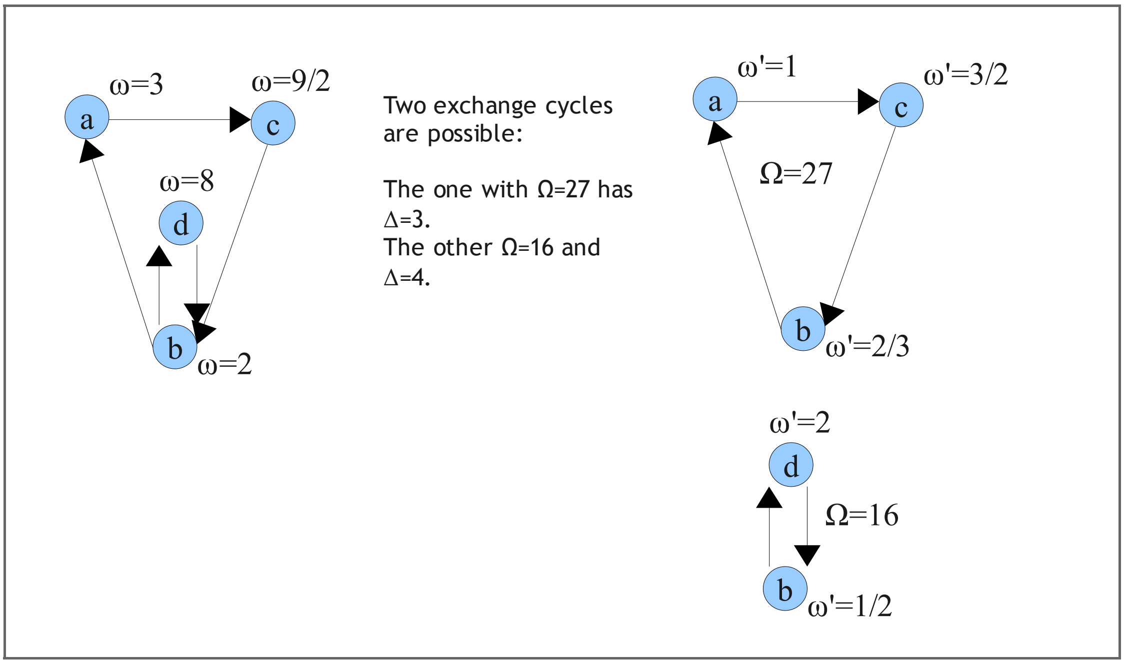
If maximizing Ω and Δ are equivalent on bilateral exchanges, they are distinct when applied to non-bilateral exchanges, due to the variable n of [a].

1 If prices are $p_b$ for the buyer and $p_s$ for the seller, we have $p_b=\omega_b$ and $p_s=1/\omega_s$ since the ratio provided/obtained depends on the view point; one being the inverse of the other. The compromise is $\omega_b'$ for the buyer:

$$\omega_b'=\omega_b.\Omega^{-\frac{1}{2}}=[p_b.p_s]^{\frac{1}{2}}$$

it is also the price compromise since $p_b'=\omega_b'$ and $p_b'=p_s'$.

the following example shows that competition on best Ω is not always that of the best Δ. For a new bid noted *b* that forms two cycles:




The maximisation of Δ leads to the choice of the bilateral cycle even if the trilateral cycle maximizes the collective product.

More generally, let's consider two cycles having collective products of $\Omega_1$ and $\Omega_2$ such as $\Omega_1 < \Omega_2$, and having the same number n of participants:

- If $\Omega_1 < \Omega_2 < 1$, the collective product is a loss for both cycles. We have: $\Omega_1 < \Omega_2 < 1 \Rightarrow \Omega_2^{\frac{1}{n}} < \Omega_1^{\frac{1}{n}} < 1 \Rightarrow \Delta_2 < \Delta_1$
  By maximizing Δ, we minimize Ω.
- If $1 < \Omega_1 < \Omega_2$, the collective product is a profit for both cycles. We have: $1 < \Omega_1 < \Omega_2 \Rightarrow 1 < \Omega_1^{\frac{1}{n}} < \Omega_2^{\frac{1}{n}} \Rightarrow \Delta_1 < \Delta_2$
  By maximizing Δ, we maximize Ω.
- If $\Omega_1 < 1 < \Omega_2$, By maximizing Δ, we also maximize the Ω.

For a given number n of participants, by maximizing Δ, we also maximize Ω, except when $\Omega 1 < \Omega 2 < 1$. In this case, the collective product is minimized, and collective loss maximized!

Let's consider now two cycles having the same Ω but different number of participants; by maximizing the individual product, the collective product is privatized (n is minimized) when it's a profit (Ω >1), and collective product is collectivized (n is maximized) when it's an effort (Ω <1).

In other words:

By maximizing the individual product Δ, profits are privatized, and losses socialized.

By maximizing the collective product Ω, losses are shared as well as profits, and competition improves performance as well as cooperation.

By extending economic exchanges to non-bilateral exchanges, barter makes the distinction between these fundamental drivers.

How this Δ maximisation explains worldwide markets behaviour is out of the scope of this paper, but could eventually explain why 80% of resources are allocated to 20% of the worldwide population.

# 3 Signal separation

The result of millions of years of interaction between environment and life created a narrow domain of stability where mankind has born. Due to the complexity of life, we have a limited knowledge of this domain and its frontiers, but enough to be sure the stability depends now significantly on the human activity.

If we simplify the model describing the use and reserves of natural resources of earth to a model developed by Control theory, called a "continuous time invariant linear system", we have:

$$\dot{x} = Ax(t) + Bu(t)$$

where:
x is a (n x 1) "state vector" represents the reserves of natural resources,
u is a (r x 1) "control vector" represents the use of resources,
A is the (n x n) "state matrix",
B is the (n x r) "input matrix".

Each dimension describing the quantity of a given mineral.

Control theory defines *Controllability*[2] as the ability of *u* to move the internal state *x* of the system from an initial state to any other final state in a finite time interval.

The system is controllable if the controllability matrix given by:

$$R = [B\; AB\; A^2 B \ldots A^{n-1} B]$$

has a full row rank (i.e. rank(R) = n).

An important consequence is that a necessary condition for the system to be controllable is that the dimension of the input vector *u* is at least that of the state vector *x*.

This shows the evidence that the number of independent commands required to maintain the stability of a system grows as it's complexity.

This model only describes a local behaviour; a better approximation should use non-linear functions. This would reduce the domain where stability can be maintained , and

2 Katsuhito Ogata (1997) Modern Control Engineering (3rd ed.) ISBN 0-13-227307-1

would define limits where the behaviour would become catastrophic.

The GIEC is developing far more sophisticated models to increase it's comprehension of the complexity of global warming. One of the first models they developed was considering a sphere enveloping the earth, and the contribution of each greenhouse gas in the flow of energy going in/out of this sphere. A *radiative forcing* (in watt/$m^2$, positive for warming - energy entering into a sphere surface unit) is always presented by the GIEC as the fundamental measurement of the contribution of each gas spices. But *life time* of gases in the atmosphere is also measured independently, greatly depending of their natures.

*Global Warming Potential*, used by the Kyoto protocol to convert greenhouse gases to $CO_2$ equivalent, are the result of a compromise between scientific evidences and the necessity of the financial negotiation, but not a scientific statement.

> The ecological regulation requires the *independence* of a wide diversity of signals that cannot be achieved by financial regulation.

# 4 Implementation

We describe here briefly a software to be developed that could implement the barter model presented here. It is a barter market place where:

- An owners is an external actor using the service of the market place.
- A value is represented by a couple (quality,quantity)
- A depository an organization managing the ownership of measurable goods. It defines its own measurement standards (quality), and guaranties the reality of the information it provides both to owners and the market place. It is equivalent to a central bank, or to a registry recording $CO_2$ quotas.

The market place is a server using graph theory by representing offers as nodes, and possible match between them as arcs. Clients of the market place are depositories acting according to orders of owners. The market place maintains the graph of offers acyclic.

From the view point of an owner, the exchange is made by the following steps:

- it deposits the value to a depository accepting the given quality,
- it makes a bid on the market place, bases on the stock he just deposited,
- it receives a draft agreement forming with his bid possible exchange cycles,
- it accepts the draft agreement. As soon all partners have also accepted, the property of corresponding values is exchanged by a circular permutation.

In addition:

- more than one bid can be made on a given value,
- more than one agreement can be formed from a given bid,

independently, the owner can evaluate the $\omega$ he would obtain if he was making a bid to provide one quality and require an other.

A prototype has been implemented confirming it's feasibility, and the necessity to increase:

- reliability et security required by the value of informations,
- scalability required by the heavy computing load of graph algorithms.

The effort invested on this possible solution should be in proportion of the benefit mankind could receive in exchange.

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